Indian Statistical Institute, Bangalore

B. Math. First Year, First Semester

Probability Theory: Back paper Examination

Date : 07-01-2015 Time: 3 hours

Maximum score: 100

1. Suppose V is a random variable taking values in $\{0, 1, 2, \ldots\}$ with $P(V = j) = p_j, j \geq 0$. The probability generating function of V is the function $g : (-1, 1) \to \mathbb{R}$ defined by

$$g(t) = \sum_j p_j t^j.$$

Compute the probability generating function of V, if V has Binomial distribution with parameters (n, p). [10]

2. Let X be a random variable having a probability density function given by

$$f(x) = \begin{cases} cx & \text{if } 0 \leq x \leq 1; \\ c(2-x) & \text{if } 1 < x \leq 2; \\ 0 & \text{otherwise,} \end{cases}$$

where c is a real number. Determine c. Let F be the distribution function of X. Let $U = F(X)$. Compute the distribution function and a density for U. [20]

3. Consider throwing of a die twice. Let F be the outcome of first throw and let G be the outcome of second throw. Obtain the joint distribution of $F + G$ and $F - G$. Obtain the conditional distribution of $F + G$ given $F - G = 0$. Obtain the conditional expectation of $F + G$ given $F - G = 0$. [20]

[P.T.O.]

4. In a quiz contest so far you have won Rs10000. For the next question, you need to choose the answer out of four options. Now you have two choices, either you can quit or you may continue playing. If you quit, you retain Rs10,000/. If you decide to continue and choose the correct answer, your prize money increases to Rs50,000/, on the other hand if you make a wrong choice, you get nothing. What would be the correct choice if you want to maximize your expected gain, assuming that you don't know the answer to the next question and you would only be making a random choice, assigning equal probability to all four options. [10]

5. Suppose P is a Possion random variable with parameter $\lambda > 0$. Compute its moment generating function. Use this to obtain the expectation and variance of P. [10]

6. Suppose Y is a discrete random variable. Show that if Y has finite second moment then it has finite finite first moment. [15]

7. State and prove weak law of large numbers for i.i.d. random variables with finite second moment. (You may assume Chebyshev's inequality) [15]